Financial distress and real economic activity in Lithuania: a Granger causality test based on MF-VAR

Ieva Mikaliunaite · Andrea Cipollini

Abstract  In this study we, first, extend the monthly Financial Stress Index (FSI) for Lithuania computed by ECB (see Duprey et al. (2017)) to a high-frequency (daily) horizon and we also include the banking sector among its constituents (beyond bond, equity, foreign exchange markets). The empirical results suggest no evidence of Granger causality between the monthly FSI index (developed by Duprey et al. (2017)) and monthly industrial production growth. On the contrary, a Granger causality test applied to a VAR using mixed frequency data characterised by a large mismatch in sampling frequencies of the series involved (i.e. daily vs monthly), suggests that the daily Lithuanian FSI has a predictive power for monthly Lithuanian IP growth for the full sample period (October 2001 – December 2016), but not vice versa. Full sample results are confirmed by rolling-window analysis.

Riassunto  In questo studio, in primo luogo, estendiamo l’indice mensile di stress finanziario (FSI) per la Lituania calcolato dalla BCE (cfr. Duprey et al. (2017)) ad un orizzonte ad alta frequenza (giornaliero) e includiamo anche il settore bancario tra i suoi costituenti (oltre i mercati obbligazionari, azionari, dei cambi). I risultati empirici non suggeriscono alcuna evidenza di causalità di Granger tra l’indice FSI mensile (sviluppato da Duprey et al. (2017)) e la crescita mensile della produzione industriale. Al contrario, un test di causalità Granger applicato a un VAR utilizzando dati a frequenza mista caratterizzati da una grande discrepanza nelle frequenze di campionamento della serie coinvolte (cioè giornaliero vs mensile), suggerisce
che l’FSI lituano giornaliero ha un potere predittivo per la crescita mensile della produzione industriale lituana per l’intero periodo di campionamento (ottobre 2001 - dicembre 2016), ma non viceversa. I risultati completi del campione sono confermati dall’analisi rolling-window.

**Parole chiave** Granger Causality - Financial Stress Index - Mixed Frequency Data - MIDAS

1 Introduction

Measuring financial stress has become more prominent since the global financial crisis. Central banks and international organizations have constructed financial stress indexes (FSI) in order to detect signs of financial stress in the whole financial system and to monitor the state of financial stability. Recently, European Central Bank has introduced a monthly Country-Level Index of Financial Stress (CLIFS) for each of the 27 European Union countries (Duprey et al. (2017)), including Lithuania.\(^1\) This index is constructed by aggregating six financial distress measures, representing the uncertainty and sharp corrections in market prices, that covers only three financial market sectors: bond, stock and foreign exchange markets.

In this paper, first, we seek to improve a monthly Country-Level Index of Financial Stress (CLIFS) for Lithuania (by Duprey et al. (2017)) along two dimensions. First, we extend a monthly ECB financial stress index to a high frequency (daily) horizon and, then, by arguing an important role played by Scandinavian commercial banks in the Lithuanian financial sector development, we include the banking sector among its constituents (beyond bond, equity, foreign exchange markets).\(^2\) More specifically, Lithuanian financial sector is dominated by Scandinavian-owned commercial banks. The three largest banks in Lithuania – SEB, Swedbank and DnB – to a significant extent have contributed to the Lithuanian economic growth over the 2000-2007 period.\(^3\) In particular, the growth was fuelled by cheap credit provided by the banks that drove up domestic demand and led to the formation of a ‘bubble’ in the Lithuanian real estate market. In 2009 the domestic real estate ‘bubble’ burst and the global financial crisis have led Lithuania into the biggest recession since the independence period. Gross Domestic Product of Lithuania fell -15% in 2009 compared to previous year.\(^4\)

---

\(^1\) Financial stress indexes were introduced for US (Hakkio et al. (2009); Kliesen et al. (2010); Brave and Butters (2011); Oet et al. (2011)), Canada (Illing and Liu (2006)), major advanced and emerging counties (Cardarelli et al. (2011); Balakrishnan et al. (2011), respectively) and Eurozone as a whole (Hollo et al. (2012) among others.

\(^2\) Also the Bank of England paper by Chatterjee et al. (2017) introduce a FSI for the United Kingdom by extending the CLIFS index by Duprey et al. (2017). The authors incorporate three additional sub-indexes that represent stress in corporate bond, money and housing markets.

\(^3\) Since 2017 the Baltic operations of DnB and Nordea banks were merged to a new bank - Luminor.

\(^4\) While we emphasize the dependence of the Lithuanian financial system from foreign banks, a recent study by Rubio and Comunale (2018) emphasize the vulnerability of the Lithuanian housing markets to Euro area common shock, given that Lithuania has variable-rate mortgages and a higher LTV cap than its European partners.
In the second step of the analysis, I contribute to empirical literature exploring the linkages between financial stress and real economic activity. Some studies have recently focused on financial uncertainty as a possible driver of the US business cycle. More specifically, the study of Bloom (2009) obtain an indicator of financial uncertainty by aggregating firm specific financial uncertainty and assess its impact on real economic activity (employment and industrial production). Ludvigson et al. (2015) extract financial uncertainty as a latent variable from a dynamic factor model fitted to a large dataset of financial time series and they assess the impact on real economic activity, proxied by log of real industrial production. Gilchrist et al. (2014) provide a micro and macro based analysis showing that financial frictions are an important part of the mechanism through which uncertainty shocks affect the economy. The authors at macro-level, using a structural vector autoregressive (SVAR) model assess the interactions between uncertainty, credit spreads, and economic activity. The results show that the interaction between financial frictions (proxied by credit spreads) and uncertainty are important to assess how fluctuations in the latter are propagated to the real economy. Unanticipated increases in uncertainty imply a rise in credit spreads, leading to a decline in real GDP that is driven primarily by the protracted drop in the investment component of aggregate spending. In contrast, shocks to financial disturbances (orthogonal to credit spreads) have a large effect on economic activity.

Moreover, a number of empirical studies find that an increase in financial stress has an adverse impact on the overall economic activity. Hakkio et al. (2009) show that an increase in financial stress leads to persistent business cycle downturns. More recently, Chau and Deesomsak (2014) show that the lagged values of financial distress have a significant predictive power for the overall U.S. economic activity. As for the Eurozone, Hollo et al. (2012) find that an increase in the financial distress, proxied by a CISS index, leads to a collapse in industrial production (only for values of the index above a threshold). More recently, Kremer (2016) shows that the CISS index Granger causes EU real GDP growth.

While the aforementioned studies are based on a common frequency dataset, in this paper, we investigate a causal relationship between a daily financial stress index for Lithuania and a monthly Lithuanian industrial production growth. For this purpose, we use a Granger (non-) causality test applied to a mixed-frequency VAR. As argued by Ghysels et al. (2016), the use of mixed-frequency data allows a more accurate analysis of the causal patterns than a test based on traditional common-frequency data. In addition, given that the mixed-frequency VAR is characterised by a large mismatch in frequencies of the series involved (e.g. daily vs monthly), we apply the Granger causality test developed by Götz et al. (2016) and by Ghysels et al. (2018).

Our findings are in line with Cardarelli et al. (2011) suggesting that banking sector stress tends to be associated with larger negative IP growth, than stress episodes related only with bond, equity and foreign exchange sectors. More specifically, in a common-frequency framework we find that the inclusion of the banking sector related stress in the financial stress index for Lithuania provides more information about the future path of IP growth in Lithuania. Finally,
we show that a proposed daily financial stress index for Lithuania is a better predictor for a future path of a monthly industrial production growth than a monthly CLIFS index of ECB.

Our study is structured as follows. Section 2 describes the recent stylized facts about the Lithuanian financial system and the real economic activity. Section 3 discusses the empirical literature on Financial Stress Index. Section 4 describes our contribution to the construction of FSI for Lithuania. Section 5 describes the Granger causality test based on the MF-VAR. Section 6 discusses the empirical evidence and section 7 concludes.

2 Stylized Facts for Lithuania

The development of the Lithuanian financial system over the period 2001-2016, measured by financial system’s asset-to-GDP ratio, is shown in Figure 1.5 The financial system’s growth (from 35.7% of GDP in 2001 to 84.5% of GDP in 2016) was mainly driven by the banking sector expansion. In fact, the asset-to-GDP ratio of the banking sector increased from 31.4% in 2001 to 66.7% in 2016.

In total, there are six banks and eight foreign bank branches operating in Lithuanian banking sector. Figure 2 shows that the Lithuanian banking sector is dominated by three Scandinavian-owned commercial banks: Swedish SEB bank and Swedbank, and Norwegian DnB bank. In particular, the assets of the three Scandinavian banks constitute around 73% of the total banking sector assets in 2016. Due to the high concentration in the Lithuanian banking sector the three major banks produce a massive systemic effect on the Lithuanian financial sector.

In the eight-year period from 2000 to 2007 the Lithuanian economy experienced one of the highest economic growth rates within the European Union. As suggested by Kuodis et al. (2009), the growth was fuelled by easy accesses to cheap credit provided by the large Scandinavian-owned commercial banks. On the other hand, the cheap credit and high income expectations gave a strong boost to the construction sector, which led to a formation of a “bubble” in the Lithuanian real estate market.

In 2009 Lithuania went into the biggest recession since the independence period (i.e. since 1990). In the first quarter of 2009 the Lithuanian industrial production fell more than 25% compared to the same period in the previous year.6 Lithuanian economy was hit by a double-crisis: external one, caused by global financial crisis and internal one, caused by a strong decline in the domestic demand (due to households and firms facing difficulties in meeting their liabilities to credit institutions).

At the end of 2011, the fifth largest Lithuanian bank, SNORAS bank, went bankrupt. According to the Bank of Lithuania data, SNORAS bank constituted 6.2% of total banking sector loans and 13.0% of deposits. In February 2013, another Lithuanian bank - Ukio bankas - went bankrupt. The bank was not a major credit provider, however, it was the fourth in terms

---

of deposit holdings. Nevertheless, the suspension of several institutions did not cause any major turbulence in the financial system.

At the beginning of 2013, Lithuania’s economy bounced back and grew at one of the fastest rates in the EU. However, in the following years the economic growth slowed down due to the uncertainty caused by the Russia-Ukraine conflict and import restriction to Russia in 2014. At the beginning of 2016, the distress in the banking sector increased due to the concerns regarding the real estate sector in Sweden. While we emphasize the dependence of the Lithuanian financial system from foreign banks, a recent study by Rubio and Comunale (2018) emphasize the vulnerability of the Lithuanian housing markets to Euro area common shock, given that Lithuania has variable-rate mortgages and a higher LTV cap than its European partners.

3 Financial Stress Index

Since the start of the global financial crisis a number of studies have developed indices of financial stress (FSI) which are used to measure the vulnerabilities in the financial system. The first study is the one by Illing and Liu (2006) introducing a FSI for the Canadian financial system combining (through principal component analysis) information on 11 financial market series representative of the banking, foreign exchange, debt and equity markets. The IMF study by Cardarelli et al. (2011) introduces FSIs for 17 advanced economies. Through variance-equal weighting method, the authors combine information on three financial market segments: banking, securities markets and foreign currency. Similarly, the IMF study by Balakrishnan et al. (2011) uses the methodology of Cardarelli et al. (2011) to construct a financial stress index for emerging countries.

As for the US, the first study to provide an index monitoring stress in the financial markets is the Kansas City FSI developed by Hakkio et al. (2009). The authors use a principal component analysis to combine 11 indicators, representing the key features of financial stress in the US financial system, into an overall index. Kliesen et al. (2010) propose a St. Louis Fed Financial Stress Index by using 18 weekly data series. Brave and Butters (2011) introduce the National Financial Conditions Index (NFCl), monitoring the financial conditions in banking sector, money, debt and equity markets. The authors show that the NFCl is useful in forecasting growth in US gross domestic product and business investment from two to four quarters ahead. Finally, another FSI index is the Cleveland Fed’s Financial Stress Index, developed by Oet et al. (2011). The index uses daily data collected from four financial market

---

7 By using a variance-equal weighting method each component is computed as a deviation from its mean and weighted by the inverse of its variance (Balakrishnan et al. (2011)).
8 Chen et al. (2014) examine the link between Kansas City FSI and oil prices. Index is available at: https://www.kansascityfed.org/research/indicatorsdata/kcfsi.
9 Index is available at: https://fred.stlouisfed.org/series/STLFSI.
10 Index is available at: https://www.chicagofed.org/research/data/nfci/background.
11 Nazlioglu et al. (2015) analyse a volatility transmission between oil prices and Cleveland FSI. Index was discontinued in May 2016.
sectors — credit, foreign exchange, equity, and interbank markets, which are aggregated into the composite indicator by applying time-varying credit weights.

The European Central Bank (ECB) periodically publishes a weekly Composite Indicator for Systemic Stress (CISS), developed by Hollo et al. (2012). The CISS index is constructed by aggregating 15 raw indicators of financial stress capturing the developments in five sectors of the Euro area: the money, foreign exchange, equity, bond and non-bank financial intermediaries securities markets.

The ECB database also provides a monthly Country-Level indicator of financial stress, CLIFS, developed by Duprey et al. (2017) for each Eurozone country, including Lithuania. The methodology for the construction of the CLIFS index is similar to the one suggested by Hollo et al. (2012) for the CISS index. However, the CLIFS captures systemic stress only in three financial market segments: equity, long term bonds and foreign currency markets.

4 Construction of Financial Stress Index for Lithuania

In this section we describe the construction of the daily financial stress index for Lithuania. More specifically, the construction involves three steps: section 4.1 describes the financial time series selected for each sub-sector; section 4.2 explains the methodology used for the transformation of market specific stress indicators; section 4.3 describes how individual indicators are aggregated into the final index.

4.1 Data and market indicators

Our daily FSI for Lithuania is constructed by using 12 market specific indicators (see Table 1). Similarly to Duprey et al. (2017), we use:

a) a Lithuanian stock market index - OMX Vilnius (OMXV) - for the equity market,

b) a 10-year government bond yields - to monitor stress in the bond market,

c) and compute a daily real effective exchange rate for Lithuania - in order to monitor stress in the foreign exchange market.

Moreover, beyond the three financial markets considered by Duprey et al. (2017), we also consider the stress in a banking sector, which we proxy by the stock prices of the three major banks operating in Lithuania: Swedbank, DnB and SEB bank. The construction, data sources and the time spans of indicators are described below.

---

12 CISS index is available at: https://sdw.ecb.europa.eu/browse.do?node=9689686.
14 As argued by Duprey et al. (2017), other sectors are not considered because the availability of data capturing stress in 27 countries is limited both in the time and cross-sectional dimension.
15 Duprey et al. (2017) capture the stress in the banking sector by using the bank stock price indices from Datastream. However, it is not available for Lithuania.
4.1.1 Bond market

To measure a distress in Lithuanian bond market we collect a daily 10-year Lithuanian government bond yields for the period ranging from 01/10/2001 to 30/12/2016. The 10-year Lithuanian government bond yields \( R_{10,LT,t} \) in the real terms are given by:

\[
r_{R10,LT,t} = R_{10,LT,t} - \frac{CPI_{LT,t} - CPI_{LT,t-12}}{CPI_{LT,t-12}} \times 100
\]

where \( R_{10,LT,t} \) is the nominal 10-year government bond yield and \( CPI_{LT,t} \) is the Consumer Price Index for Lithuania; \( t \) denotes days and \( t \) indicates months. Since the CPI is available only on monthly frequency, we simply interpolate the monthly CPI to a daily frequency. Then, we estimate two components of the bond market sub-index:

(i) daily realized volatility \( (V_{R10,LT,t}) \) obtained from the absolute daily changes in the real 10-year Lithuanian government bond yields \( (r_{R10,LT,t}) \). In line with Duprey et al. (2017) we standardize the changes in the real 10-year Lithuanian government bond yields \( (chr_{R10,LT,t}) \) through a 10 year rolling standard deviation (i.e. the window size is set equal to 2520 working days):

\[
\begin{cases}
chr_{R10,LT,t} = r_{R10,LT,t} - r_{R10,LT,t-1} \\
chr_{R10,LT,t} = \frac{chr_{R10,LT,t}}{\sigma_{chr_{R10,LT,t},10years}} \\
V_{R10,LT,t} = \left| chr_{R10,LT,t} \right|
\end{cases}
\]

(ii) cumulative difference \( (CDIFF_t) \) computed as a maximum increase in Lithuanian real government bond spread over a two-year rolling window (i.e. over the previous \( T = 2 \) years). In particular the real government bond spread with respect to Germany \( (r\text{\textit{Spread}}_{10,DE,t}) \) is given by:\[16\]

\[
\begin{cases}
r\text{\textit{Spread}}_t = r_{R10,LT,t} - r_{R10,DE,t} \\
CDIFF_t = r\text{\textit{Spread}}_t - \min_{i=0,...,T}(r\text{\textit{Spread}}_{t-i})
\end{cases}
\]

4.1.2 Equity market

The Lithuanian stock market index, OMX Vilnius (OMXV), includes all the stocks listed on the main and secondary lists on the Vilnius Stock Exchange. The stock market index in real terms is given as:\[17\]

\[16\] The daily data on Lithuania bond yields is obtained as the difference of the daily spread with German 10 year government bond yield available from Ycharts: https://ycharts.com/indicators/lithuaniagermany_10_year_bond_spread, and the daily 10-year German government bond yields available from Bundesbank database. Then, we use the monthly CPI for Lithuania and for Germany, available from OECD to convert the nominal yields into real term.

\[17\] Note: CPI is available only on monthly frequency, therefore, we simply interpolate it to a daily frequency.
Similarly to the bond market, we follow the suggestion of Duprey et al. (2017) and focus on:

(i) \textit{daily realized volatility} \( (VOMXV_t) \) obtained from the absolute daily log stock market returns:

\[
\ln rOMXV_t = \log(rOMXV_t) - \log(rOMXV_{t-1})
\]

\[
\ln rOMXV_t = \sigma_{\ln rOMXV_{t-10\text{years}}}
\]

\[
VOMXV_t = \left| \ln rOMXV_t \right|
\]

where the returns are standardized by using a 10 year rolling window standard deviation.

(ii) \textit{cumulative maximum loss} \( (CMAX_t) \), estimated by comparing the value of \( rOMXV_t \) at day \( t \) with its maximum value over the previous \( T \) periods \( (T = 2 \text{ years}, 507 \text{ days}) \):

\[
CMAX_t = 1 - \frac{rOMXV_t}{\max_{i=0,1,...,T}(rOMXV_{t-i})}
\]

where the backward rolling window is fixed for the first 2 years \( (04/01/2000 - 31/12/2001) \).

\[\textbf{4.1.3 Foreign exchange market}\]

The Lithuanian foreign exchange market dynamics is monitored by focusing on the \textit{real effective exchange rate}, REER. However, the Bank of International Settlements (BIS) and the ECB Statistical Data Warehouse (SDW) publish the REER for Lithuania only at low frequencies (monthly, quarterly or annual). Therefore, we construct a \textit{daily} REER.

Unlike the bilateral exchange rate that involves two currencies, the effective exchange rate is an index that describes the strength of a currency relative to a basket of other currencies. In particular, we calculate the REER for Lithuania as the geometric weighted average of bilateral nominal exchange rates of litas vis-à-vis the currency of the major trading partners. The major trading partners are: the whole Eurozone (euro), Estonia (kroon), Latvia (lats), China (yuan renminbi), Czech Republic (koruna), Denmark (krone), Japan (yen), Norway (krone), Poland (złoty), Russia (ruble), Sweden (krona), Turkey (lira), United Kingdom (pound sterling), United States of America (dollar).\(^{18}\) The nominal bilateral currencies are then converted in purchasing power of Lithuanian consumers by using the country specific consumer price index (CPI) (Schmitz et al., 2013):

\[
REER^f = \prod_{i=1}^{N} \left( \frac{\hat{e}^{e}_{LT,i} CPI_{LT}^i}{CPI_i^i} \right)^{w_i}
\]

\(^{18}\) Estonia and Latvia joined the Eurozone in 2011 and 2014, respectively.
where $N$ is the number of major trading partner countries; $e_{LT,i}^t$ is a bilateral exchange rate of the litas vis-à-vis the currency of partner country $i$; $w_i$ is the trade weight assigned to the currency of trading partner; the CPI for Lithuania and for the partner country $i$ are $CPI_{LT}^t$ and $CPI_i^t$ respectively.\(^\text{19}\) The weights assigned to the major trading partners are shown in Table 2. The 11 Eurozone countries and other 13 major trading partner countries cover 91.4% of Lithuanian total trade in the period 2008 – 2010. The weights are adjusted considering 91.4% to be the total trade (see second row of Table 2). For the comparison, Figure 3 shows that our daily REER (in figure aggregated to monthly frequency) is almost identical to the monthly REER from BIS.

Then, similarly to bond and stock market, the stress in foreign exchange market is monitored by measuring the following two components:

(i) \textit{daily realized volatility} ($VREER_t$) computed as the absolute value of daily growth rate of real effective exchange rate. We divide the growth rate by a 10 years rolling standard deviation (with the window set equal to 2520 working days):

\begin{align}
\ln REER_t &= \log(REER_t) - \log(REER_{t-1}) \\
\ln \tilde{REER}_t &= \ln REER_t - \sigma_{\ln {\tilde{REER}}_{t-10years}} \\
VREER_t &= \frac{\ln \tilde{REER}_t}{\sigma_{\ln {\tilde{REER}}_{t-10years}}} \\
\end{align}

(ii) \textit{cumulative change} (CUMUL) of REER over six months ($i = 6$ months, or 126 working days):

$$CUMUL_t = |REER_t - REER_{t-i}|$$

\subsection*{4.1.4 Banking sector}

In order to measure the stress in the Lithuanian banking sector, we monitor the stock prices of the three major Scandinavian banks: the Norwegian DnB bank and the two Swedish banks – Swedbank and SEB. In particular, the banking sector sub-index consists of six components. For each of the bank we estimate two components:\(^\text{20}\)

(i) \textit{daily realized volatility} of the idiosyncratic part of the bank stock price returns ($VBKS_{Bi,t}$). The idiosyncratic component ($\epsilon_{Bi,t}$) is the estimated residual from a regression of the bank specific real stock price return ($\ln BS_{Bi,t}$) on the real total stock market index ($\ln rSX_{c,t}$):

\[^{19}\] The bilateral exchange rates for litas and its major trading partners are collected from the Bank of Lithuania (BoL); the CPI data is taken from OECD and the trading weights from BIS. Note: CPI is available only on monthly frequency, therefore, we simply interpolate it to a daily frequency.

\[^{20}\] We follow the methodology by Duprey et al. (2017) for the components’ construction. However, note that the authors do not include the banking sector in the CLIFS for Lithuania.
Ieva Mikaliunaite, Andrea Cipollini

\[
\begin{align*}
\ln BKS_{B,t} &= \log(rBKS_{B,t}) - \log(rBKS_{B,t-1}) \\
\ln BKS_{B,t} &= \beta_{B,t} \times \ln rSX_{c,t} + \epsilon_{B,t} \\
\epsilon_{B,t} &= \frac{\tilde{\epsilon}_{B,t}}{\sigma_{B,t-10years}} \\
V BKS_{B,t} &= |\epsilon_{B,t}|
\end{align*}
\]  

(10)

where the regression is estimated by using a rolling window of two years (fixed for the first two years). The stock market indexes (indexed by \(c = \text{OMXS30, OBX} \)) of Sweden stock market (OMXS30) and Norwegian stock market (OBX) and bank specific stock prices (indexed by \(B = \text{Swedbank, SEB, DnB} \)) are converted in real terms by using the CPI for the related countries, respectively, as:

\[
\frac{sX_{c,t}}{CPI_{c,t}} \quad \text{and} \quad \frac{BKS_{B,t}}{CPI_{c,t}}.
\]

(ii) cumulative maximum loss of bank stock prices (CMAXB) for each bank is estimated by comparing the value of \(rBKS_B\) at time \(t\) with its maximum value over the previous \(T\) periods (\(T = 2\) years, 502 working days):

\[
CBKS_{B,t} = 1 - \frac{rBKS_{B,t}}{\max_{i=0,1,...,T}(rBKS_{B,t-i})}
\]  

(11)

4.2 Transformation of raw stress indicators

In order to aggregate the twelve individual stress indicators into a single financial stress index, firstly, we need to standardize each indicator to have a common unit. Following Hollo et al. (2012) and Duprey et al. (2017), the standardization of stress indicators is based on empirical cumulative distribution function (CDF). This method of standardization consists in converting the six financial stress indicators into new series which are unit-free and ranging between 0 and 1. By this procedure, in the first step, the values of individual stress indicator are ranked and then divided by the total number of observations \((n)\). The rank of 1 is assigned to the minimum value in the sample and \(n\) to a maximum.

The empirical CDF is computed as:

\[
z_n = F_n(x_n) = \begin{cases} 
\frac{z}{n} & \text{for } x_{[r]} \leq x_t \leq x_{[r+1]} \\
1 & \text{for } x_n \geq x_{[n]}
\end{cases}
\]  

(12)

21 The OMXS30 is a stock market index for the Stockholm Stock Exchange that consists of the 30 most traded stock classes (including SEB bank and Swedbank stocks). The OBX Index is a stock market index which lists twenty-five most liquid companies (including DnB bank) of the Oslo Stock Exchange in Norway. The data on Swedbank and SEB bank stock prices as well as OMXS30 index are from NASDAQ database. The data on DnB bank stock prices is collected from DnB database and the OBX index from Oslo Bors database. Note: CPI is available only on monthly frequency, therefore, we simply interpolate it to a daily frequency.
where \( r = \{1, 2, \ldots, n-1\} \) is a rank number, \( n \) the total number of observations in the sample.\(^\text{22}\)

The CDF is computed over an initial window of 10 years, after this period, the transformation is applied recursively over expanding samples with one new observation added at a time (keeping the ranks of previous observations fixed).

4.3 Aggregation

Once the twelve stress indicators have been transformed, we aggregate them into the final FSI. The aggregation consists in two steps. In the first step, the transformed individual stress indicators capturing stress in a specific financial market are combined (by arithmetic average) to obtain four sub-indexes: the bond market sub-index \( (S_{\text{Bond}}) \), the stock market sub-index \( (S_{\text{Eq}}) \), the foreign exchange market sub-index \( (S_{\text{FX}}) \) and the banking sector sub-index \( (S_{\text{Bank}}) \):

\[
\begin{align*}
S_{\text{Bond}, t} &= \frac{Vr_{\text{R}0} + CDIFF_{t}}{10} \\
S_{\text{Eq}, t} &= \frac{VOM_{X} + CMAX_{t}}{2} \\
S_{\text{FX}, t} &= \frac{VREER_{t} + CUMUL_{t}}{2} \\
S_{\text{Bank}, t} &= \frac{VBKS_{\text{Sved}, t} + CBKS_{\text{Sved}, t} + VBKS_{\text{SEB}, t} + CBKS_{\text{SEB}, t} + VBKS_{\text{DnB}, t} + CBKS_{\text{DnB}, t}}{6}
\end{align*}
\]

Once the sub-indices are computed, we aggregate them into the final FSI by using the approach based on the portfolio theory suggested by Hollo et al. (2012) for the CISS index construction and used more recently by Louzis and Vouldis (2013), Johansson and Bonthron (2013) and by Duprey et al. (2017)). Therefore, the FSI for Lithuania is computed as follows:

\[
FSI_{t} = (w \times s_{t})C_{t}(w \times s_{t})'
\]

where \( s_{t} = (S_{\text{Bond}, t}, S_{\text{Eq}, t}, S_{\text{FX}, t}, S_{\text{Bank}, t}) \) is a vector of the sub-indexes, \( C_{t} \) is the matrix of time-varying cross-correlation coefficients between the four sub-indexes and \( w \) is a sub-index weight. Similarly to Duprey et al. (2017), we give the same weight to each sub-index \( (w = \frac{1}{4}) \).\(^\text{23}\)

The portfolio based approach allows taking into account the systemic co-movement across the financial market segments through time-varying cross-correlations between the sub-indexes. The stronger is the correlation of financial stress across the sub-indexes, the more weight is attributed to the FSI. The time-varying cross-correlation \( \rho_{i,j,t} \) between sub-indexes \( i \) and \( j \) is estimated recursively using the exponentially weighted moving averages (EWMA) method. In particular, the covariances \( (\sigma_{i,j,t}) \) and volatilities \( (\sigma_{i,t}^{2}) \) are estimated as follows:

\(^\text{22}\) If the same value of \( x \) occurs more than once, the rank number assigned to each of the observations is given as the average of rankings involved.

\(^\text{23}\) Hollo et al. (2012) estimate the weights of the sub-indexes in the CISS index by using a bivariate linear VAR and, then, by computing the cumulated impulse response of industrial production growth to a one standard deviation shock to a sub-sector index. However, the authors find that the differences between the CISS computed with impulse response based weights and the one with equal weights are not large.
\begin{align*}
\sigma_{i,j,t} &= \lambda \sigma_{i,j,t-1} + (1 - \lambda) \tilde{S}_{i,t} \tilde{S}_{j,t} \\
\sigma^2_{i,t} &= \lambda \sigma^2_{i,t-1} + (1 - \lambda) \tilde{S}^2_{i,t} \\
\rho_{i,j,t} &= \frac{\sigma_{i,j,t}}{\sigma_{i,t} \sigma_{j,t}} \\
C_i &= \begin{pmatrix}
1 & \rho_{\text{Eq,Bond},t} & \rho_{\text{FX,Eq},t} & \rho_{\text{Bank,Eq},t} \\
\rho_{\text{Bond,Eq},t} & 1 & \rho_{\text{FX,Bond},t} & \rho_{\text{Bank,Bond},t} \\
\rho_{\text{Bond,FX},t} & \rho_{\text{Eq,FX},t} & 1 & \rho_{\text{Bank,FX},t} \\
\rho_{\text{Bond,Bank},t} & \rho_{\text{Eq,Bank},t} & \rho_{\text{FX,Bank},t} & 1
\end{pmatrix}
\end{align*}

where \( i,j = \{\text{Bond, Eq, FX, Bank}\}, i \neq j \), with \( \tilde{S}_{i,t} = (S_{i,t} - 0.5) \) denoting demeaned sub-indexes obtained by subtracting their theoretical median value (i.e. 0.5). In line with Duprey et al. (2017), we keep the smoothing parameter \( \lambda = 0.85 \) constant. The initial values for the covariance and the volatilities (for \( t = 1 \) which is associated with 2/10/2001) are set equal to the corresponding average values over the two years (i.e. the period running from 2/10/2001 to 30/9/2002).

4.4 Daily financial stress index for Lithuania

The evolution the sub-indexes used for Lithuanian financial stress index construction, over the period 2/10/2001 – 30/12/2016, is displayed in Figure 4. Figure 4 shows that the financial distress in bond, equity, foreign exchange and banking sectors reaches the peak during the period of global financial crisis. In particular, the stock market sub-index peaks in October 2008, the foreign exchange sub-index in January 2009, the banking sub-index in March 2009 and bond sub-index in June 2009.

Figure 5 shows the contribution of each sub-index to the overall distress in the Lithuanian financial system. The contribution of each sub-index increases during the global financial crisis (during mid-2007 – mid-2009) and the major contributor is the banking sector. As expected, during the European sovereign debt crisis period (beginning of 2011 – 2012) the bond market sub-index is the main contributing factor to financial stress. It is also worth noting that the foreign exchange sub-index results as the major contributor to the overall stress in Lithuania in 2015. In particular, distress in the Lithuanian foreign exchange market has increased at the end-2014, when the Russian economy was in a downturn due to the fall in the oil prices and the Russia-Ukraine conflict.

Figure 6 shows the time-varying correlations between the four sub-indices, which quantifies the systemic risk of the Lithuanian financial system. The relatively high correlation coefficient between each pair of market sub-indices associated with relatively high values of all sub-indices is observed over the period from December 2008 to September 2009.

The total daily financial stress index for Lithuania is presented in Figure 7. Figure 7 shows that the Lithuanian financial system did not experience high levels of financial stress over the
period 2001-2007. However, the index starts to increase at the beginning of 2008 and reaches a peak right after the collapse of the US investment bank Lehman Brothers. Although the financial distress slightly diminishes at the end of 2009, the rising concerns regarding the sustainability of sovereign debt in the some Eurozone countries (Greece and later Italy, among others) leads to high values of the stress index in May 2010 and over September - October 2011. Furthermore, Figure 7 shows that the failure of two domestic banks: **SNORAS** (in November 2011), which was the third largest bank by deposits and the fifth largest by assets and of **Ukio bankas** (in February 2013), did not affect the stability of the entire financial system.

Finally, Figure 8 compares our *daily* FSI with an alternative *monthly* financial stress index for Lithuania byDuprey et al. (2017), which is available at ECB database. Note, that for a more straightforward comparison, for the moment, we aggregate our daily FSI to a monthly frequency. Figure 8 shows that the two indexes peaks during the GFC. However, while our FSI peaks in the beginning of 2009, the ECB index reach the highest stress level in the mid-2009.

---

### 5 Mixed Frequency Granger (non-) Causality test

There is an important link between financial stress and the real sector. A number of empirical studies find that increase in financial stress, measured by a financial stress index, can produce substantial spillovers and have significant effects on the real economy.\(^{24}\) Hollo et al. (2012) find that in the high-stress regimes the increase in the EZ financial distress leads to a collapse in industrial production, while in the low-stress regime it does not have any statistically significant impact. More recently, Kremer (2016) shows that the CISS index Granger cause EU real GDP growth. While the aforementioned studies are based on a common low frequency dataset, in this section, I investigate a causal relationship between a *daily* financial stress index for Lithuania (constructed in section 4) and a *monthly* Lithuanian industrial production growth.

#### 5.1 Mixed Frequency VAR

Consider two time series sampled at different frequencies: a low-frequency series \(x_L\) and a high-frequency series \(x_H\). A high-frequency series is observed \(m\) times during a low-frequency period \(t\). According to Ghysels (2016), the mixed frequency VAR model can deal either with a case of a small \(m\) (e.g. when the series are sampled at quarterly/annual or weekly/daily frequency), or with a case of a large \(m\) (e.g. when the series are sampled at daily/monthly or weekly/quarterly frequency). In this paper, we focus on the large \(m\) case: one series is sampled at *monthly* and the other one at *daily* frequency.

\(^{24}\) For instance, when financial markets suffer from high distress increased uncertainty about asset value decreases the value of collateral. As the consequence, shocks affecting the creditworthiness lead to increased swings in output. At the same time, economic activity is affected by the fact that bank capital is eroded, which forces banks to deleverage and decrease the lending to businesses.
In MF-VAR all observations of period $t$ (i.e. high and low frequency observations) are stacked into a column vector by treating the $m$ observations of the high-frequency series as if they were distinct endogenous variables. Let $x_H(t, 1)$ be the first high-frequency observation of $x_H$ in low frequency period $t$ (e.g. the first daily observation of the month $t$), a $x_H(t, 2)$ – the second, and $x_H(t, m)$ – the last one. Consider a high-frequency vector in $t$-period as $[x_H(t, 1), x_H(t, 2), x_H(t, j), \ldots, x_H(t, m)]'$. Then, a mixed frequency vector with one high and one low frequency variable is denoted as $Z(t) = [x_H(t, 1)', \ldots, x_H(t, m)', x_L(t)]'$, with the dimension $K \times 1$, where $K = m + 1$.

A reduced-form vector autoregressive model with mixed-frequency data (MF-VAR(p)) is given by:

$$
\begin{align*}
\begin{pmatrix}
    x_H(t, 1) \\
    \vdots \\
    x_H(t, m) \\
    x_L(t)
\end{pmatrix}
= 
\begin{pmatrix}
    \mu_1 \\
    \vdots \\
    \mu_m \\
    \mu_{m+1}
\end{pmatrix} 
+ 
\sum_{k=1}^{p}
\begin{pmatrix}
    d_{11,k} \cdots d_{1m,k} & c_{1,k} \\
    \vdots & \vdots & \vdots \\
    d_{m1,k} \cdots d_{mm,k} & c_{m,k} \\
    b_{1,k} \cdots b_{m,k} & a_k
\end{pmatrix}
\begin{pmatrix}
    x_H(t-k, 1) \\
    \vdots \\
    x_H(t-k, m) \\
    x_L(t-k)
\end{pmatrix} 
+ 
\begin{pmatrix}
    u_H(t, 1) \\
    \vdots \\
    u_H(t, m) \\
    u_L(t)
\end{pmatrix}
\end{align*}
$$

or

$$
Z(t) = \mu + \sum_{k=1}^{p} \Gamma_k Z(t-k) + u_t
$$

The coefficients $b$’s and $c$’s capture the causality from a high-frequency variable $x_H$ to the low frequency variable $x_L$, and the causality from $x_L$ to $x_H$, respectively. More specifically, testing for Granger (non-) causality implies the following null hypothesis:

- **High-to-low (non-) causality.** $x_H$ does not Granger cause $x_L$ if and only if:

  $H_0 : b_{1,k} = \ldots = b_{m,k} = 0; \text{ for } k = 1, \ldots, p$ \hspace{1cm} (17)

- **Low-to-high (non-) causality.** $x_L$ does not Granger cause $x_H$ if and only if:

  $H_0 : c_{1,k} = \ldots = c_{m,k} = 0; \text{ for } k = 1, \ldots, p$ \hspace{1cm} (18)

### 5.2 Granger causality test

Given that the mixed-frequency VAR in section 5.1 is characterized by a large mismatch in sampling frequencies of the series involved (i.e. daily vs monthly), in this section we describe the Granger causality tests that take into account this issue. Götz et al. (2016) develop a test for a high-to-low and low-to-high Granger causality in a mixed-frequency VAR by using a Wald test. The Wald test is based on the unrestricted MF-VAR in (16). Let $\hat{\Gamma}$ denote an OLS estimates of the coefficient matrices for the lagged endogenous variables in the MF-VAR (16)

25 The parameters in $\Gamma$ can be estimated by using an OLS estimator.
and define $R$ a matrix that picks the set of coefficients of interest for Granger (non-) causality test i.e. $Rvec(\hat{\Gamma})$. Then, the Wald test statistic is constructed as:

$$\hat{\xi}_W = [Rvec(\hat{\Gamma})]'(R\hat{\Omega}R')^{-1}[Rvec(\hat{\Gamma})]$$

(19)

with

$$\hat{\Omega} = \left(W'W\right)^{-1} \otimes \hat{\Sigma}$$

where $\hat{\Sigma} = \frac{1}{T} \hat{u}'\hat{u}$ is the covariance matrix of the disturbance terms in (16) and $W$ is the regressor set. However, in a mixed-frequency model with a large $m$ an asymptotic Wald test may exhibit size distortions when the number of zero restrictions is relatively large, compared to a sample size (Götz et al. (2016); Ghysels et al. (2018)). Therefore, Götz et al. (2016) rely on bootstrap in order to draw an inference based on the Wald test.\footnote{Monte Carlo simulations in Götz et al. (2016) show that bootstrap variants of high-to-low and low-to-high causality tests improve the empirical size.}

Ghysels et al. (2018) propose a max-test only for high-to-low Granger causality case with a large number of zero restrictions. More specifically, the max-test statistic is based on $pm$ parsimonious regression models:

$$x_L(t) = \mu_i + \sum_{k=1}^{p} a_{k,i} x_L(t-k) + \beta_i x_H(t-1, m+1-i) + u_{L,i}(t),$$

(20)

where index $i \in \{1, ..., pm\}$ is in high-frequency terms and the second argument $(m+1-i)$ of $x_H$ can be less than 1 (since $i > m$ occurs when $p > 1$). Each $i^{th}$ model contains $p$ low-frequency autoregressive lags of $x_L$ and only the $i^{th}$ high-frequency lag of $x_H$. We estimate the parsimonious model $pm$ times. It is important to notice that the $\beta_i$ in parsimonious models (20) and $b$’s in unrestricted model (16) generally are not equivalent. We estimate the parameters in each $i^{th}$ model by OLS to get $\hat{\beta}_i = \{\hat{\beta}_1, ..., \hat{\beta}_{pm}\}$. Then we formulate a max-test statistic as:

$$\hat{T}_{TL} = \max_{1 \leq i \leq pm} \{ (\sqrt{T_L} \hat{\beta}_i)^2 \}$$

(21)

where $T_L$ is a low-frequency sample size. The mixed-frequency max-test statistics $\hat{T}_{TL}$ has a non-standard asymptotic distribution under the null hypothesis in (17). Therefore, we follow Ghysels et al. (2018) and rely on a simulation-based p-value.

6 Empirical Evidence

6.1 Data

We focus on the daily Lithuanian FSI (constructed in section 4) and monthly industrial production (IP) growth in Lithuania for the sample period running from October 2, 2001 to December
30, 2016, yielding a sample size of $T_L = 183$ months. In line with Hollo et al. (2012) and Duprey et al. (2017) we use a year-to-year IP growth estimated as 12th month log-difference. We use a seasonally and working day adjusted data on Lithuanian industrial production, which is available from the Lithuanian Department of Statistics.

Figure 9 plots the data. The figure suggest that Lithuanian economy has experienced a high growth up to the end-2008. With the beginning of the global financial crisis the IP shrank, decreasing for 14 consecutive months (from November 2008 to January 2010), peaking at (minus) -31.4% in April 2009.

The daily Lithuanian FSI, constructed in section 4, has a varying number of daily observations within each month, ranging from 18 to 23 observations. In order to perform a MF Granger causality test, for simplicity, we assume that the maximum amount of daily observations that is available in each month throughout the sample is 18 ($m = 18$). More specifically, the daily Lithuanian FSI series are modified as follows:

$$FSI_{H}(t,j) \quad \text{for } j = 1, \ldots, 18$$ (22)

where the last $m(t) − 18$ observations at the end of each month are disregarded (see Götz et al. (2016)). This modification gives us a dataset with $T_L = 183$ low-frequency observations, $m = 18$ and $m \times T_L = 3294$ high-frequency observations. Therefore, when setting the MF-VAR model specification, the stacked vector of endogenous variables has a dimension $K = 19$. The Phillips and Perron (1988) test for unit-root suggest that the data are stationary (at 10-percent significance level).

6.2 Granger causality tests

To test for Granger causality from the daily Lithuanian FSI ($FSI_{H}$) to monthly IP growth in Lithuania ($IP_{L}$) we focus on the last row of MF-VAR(2) specification in (16):

$$IP_{L}(t) = \mu_{19} + \sum_{k=1}^{2} a_k IP_{L}(t-k) + \sum_{i=1}^{36} b_i FSI_{H}(t - 1, 18 + 1 - i) + u_{L}(t)$$ (23)

where we regress the monthly IP growth onto a constant ($\mu_{19}$), $p = 2$ months of lagged low-frequency variable (IP growth), $pm = 36$ days of lagged high-frequency variable (FSI for Lithuania). The Wald test developed by Götz et al. (2016) consider the unrestricted model described in (16). The model test developed by Götz et al. (2016) consider the unrestricted model described in (16). The model estimate by using ordinary least squares. The p-values for the Wald test

\footnote{Note that the $x_{H}(t, 1)$ is not necessarily the first day of the month. For example, October 1, 2016 fall on Sunday. Thus, October 3, 2016 is considered as the first observation of the month $x_{H}(t, 1)$.}

\footnote{Results are available upon request.}

\footnote{The model specification is chosen according to the suggestions of Ghysels et al. (2018). More specifically, we perform a Ljung-Box Q-test to test for the absence of serial correlation in residuals of the full regression model eq. (23). The p-values for Q-test are reported in Appendix, Table 5.}
Financial distress and real economic activity in Lithuania

for the null hypothesis $H_0 : b_1 = \cdots = b_{36} = 0$ in eq. (17) are computed using 1999 bootstrap replications.\textsuperscript{30}

The max-test by Ghysels et al. (2018)) is based on parsimonious regression models:

$$IP_L(t) = \mu_i + \sum_{k=1}^{2} a_{ki} IP_L(t - k) + \beta_i FSI_H(t - 1, 18 + 1 - i) + u_{L,i}(t) \quad (24)$$

for $i = \{1, \ldots, 36\}$, where each $i^{th}$ model contains $p = 2$ months of lagged IP growth and only the $i^{th}$ daily lag of FSI for Lithuania.\textsuperscript{31} We estimate the parsimonious model 36 times. The number of parameters to estimate in the parsimonious regression model is 4 (1+2+1) against 39 (1+2+36) in the full regression model in eq.(23).\textsuperscript{32} For the robustness check we also try the different model specifications: MF-VAR(1) and MF-VAR(3).

To test for Granger causality from monthly IP growth to daily FSI for Lithuania, defined as $H_0 : c_1 = \cdots = c_{36} = 0$ in (18), we focus on an unrestricted model in (16) (see last column of the model) and we use the Wald statistics eq. (19) developed by Götz et al. (2016).

6.3 Empirical findings

6.3.1 Full sample analysis

The full sample results in Table 3 panel (A) show that the daily Lithuanian FSI has a predictive power for monthly Lithuanian IP growth for the period (October 2001 – December 2016), but not vice versa. More specifically, the p-values suggest that, for any MF-VAR model specification (with lag order $(p)$ equal to 1, 2, or 3), there is evidence of unidirectional causality from daily Lithuanian FSI to monthly IP growth in Lithuania. In fact, while both the max-test and the Wald test reject the null hypothesis that FSI does not Granger cause IP growth at 10% significance level, we cannot rejected the null hypothesis that IP growth does not Granger cause FSI.

In addition, our analysis is in line with other studies (see Hakkio et al. (2009), among others) which find that an increase in financial stress has an adverse impact on the overall economic activity. In particular, Figure 10 shows that the point estimates of the coefficients $\beta_i$ for each $i^{th}$ parsimonious model (eq. 24) used to evaluate the effect of financial stress on economic activity is negative and statistically significant (at 95% confidence interval).

Further, in order to evaluate our index, we compare the daily FSI for Lithuania with alternative financial stress indexes. Specifically, we test for Granger causality between monthly IP growth in Lithuania and (i) a daily FSI for Lithuania, constructed by taking into consideration

\textsuperscript{30} For bootstrap, we use a code provided by Götz et al. (2016).

\textsuperscript{31} Following Ghysels et al. (2018) p-values are computed based on the robust covariance matrix with 100 000 draws from an approximation to the limit distribution under non-causality.

\textsuperscript{32} We estimate 4 parameters in each $i^{th}$ parsimonious model eq. (24): (i) a constant ($a_{0i}$), (ii) two coefficients related to the lagged IP growth ($a_{1i}, a_2i$) and (iii) one coefficient related to the lagged FSI ($\beta_i$).
only three sub-indexes - bond, equity and foreign exchange (excluding banking sub-index), (ii) a monthly FSI for Lithuania (composed of 4 sub-sectors: bond, equity, foreign exchange and banking), obtained by averaging the daily FSI to a monthly frequency, and (iii) a monthly CLIFS index provided by ECB and developed Duprey et al. (2017), which is composed of only three sub-indexes - bond, equity and foreign exchange.

As for case (i), the p-values in Table 3 panel (B) suggest that a daily FSI for Lithuania, constructed by taking into consideration only three sub-indexes - bond, equity and foreign exchange (excluding banking sub-index), is also a good predictor for a monthly IP growth in Lithuania. More specifically, for any MF-VAR specification (with lag order equal to 1, 2 or 3) the daily FSI (3 sub-indexes) Granger cause a monthly IP growth in Lithuania, but not vice versa. Furthermore, the point estimates of the coefficients $\beta_i$ for each $i^{th}$ parsimonious model (eq. 24) in Figure 11 shows that an increase in financial stress in equity, bond and foreign exchange sectors leads to a slowdown in IP growth in Lithuania, although the effect is slightly smaller compared to the daily FSI that also contains the banking sector related stress sub-index.

As for case (ii), we use a common frequency VAR(1) to test for Granger causality between monthly IP growth in Lithuania and a monthly FSI for Lithuania (composed of 4 sub-sectors: bond, equity, foreign exchange and banking). The p-values in Table 3 panel (C) suggest a unidirectional causality from a monthly FSI to a monthly IP growth, since we can reject the null hypothesis of non-causality relying on an asymptotic and a bootstrap version of a Wald test. Moreover, a common frequency regression results in Table 4 panel A show that an increase in financial stress has a negative and statistically significant impact on the IP growth in Lithuania.

As for case (iii), we investigate the causal relationship between the monthly CLIFS developed by Duprey et al. (2017) and the monthly Lithuanian IP growth series.\textsuperscript{33} For this purpose, we use an asymptotic and a bootstrap Wald test for testing bi-directional Granger causality. The full sample results in Table 3 panel (D), based on VAR(2) model, suggest that we cannot reject the null hypothesis of non-causality in both directions. This empirical finding is confirmed by the common frequency regression results in Table 4 panel (B) suggesting that a CLIFS index has a negative impact on IP growth in Lithuania, although the effect is not statistically significant.

To summarize, the empirical findings suggest the importance of including the banking sector into the financial stress index (and the use of a mixed frequency dataset), otherwise the negative causal effect on IP growth would not be detected.

\textit{6.3.2 Rolling-window analysis}

We also assess if there is any evidence of changes in the causality between the daily FSI for Lithuania and monthly IP growth over the full sample period. For this purpose, we implement a rolling-window Granger causality test, fixing the window size to 84 months (i.e. seven years).\textsuperscript{34}

\textsuperscript{33} The index is composed of 3 sub-sectors, reflecting the stress in equity, bond and foreign exchange market.

\textsuperscript{34} The rolling-window analysis uses a fixed-length window, which moves sequentially from the beginning to the end of the sample period by adding one observation ahead and dropping one from the behind.
This gives a total of 100 sub-samples, where the first sub-sample covers the period from October 2001 to September 2008 and the last sub-sample covers the period from January 2010 to December 2016. In line with the full sample analysis, we use a max-test developed by Ghysels et al. (2018) and the Wald test developed by Götz et al. (2016) for each sub-sample.

Figure 12 plots the rolling window p-values for each causality test over the 100 windows (the last observation of each sub-sample period is shown on the horizontal axis). More specifically, Panel A and B show the max-test and the Wald test p-values for the null hypothesis that a daily FSI does not Granger cause the monthly IP growth in Lithuania. The p-values of the Wald test for the causality in the opposite direction are presented in the panel C of Figure 12.

In line with full sample analysis, the results suggest that we cannot reject the null hypothesis that IP growth in Lithuania does not Granger cause the financial distress in Lithuania at 10% significance level. On the contrary, we find that a daily financial stress index for Lithuania has a predictive power for monthly Lithuanian IP growth (at 10% significance level) for most of the considered sub-samples, according to the max-test and to the Wald test (see Panel A – B in Figure 12). In particular, the max-test (see panel A) confirms the causality from FSI to IP growth from April 2009 to March 2016, and the Wald test (see panel B) detects a significant causality from the January 2010 to January 2016.\footnote{Note that we report the last observation of the sub-sample period.}

Furthermore, the p-values of a rolling window analysis in Figure 13 panel (A)-(B) suggest that a daily FSI for Lithuania, constructed by aggregating only 3 sub-indexes (bond, equity and foreign exchange) is also a good predictor of monthly Lithuanian IP growth, since we can reject the null hypothesis of non-causality at 10% significance level in most of the considered sub-samples. Panel C shows that IP growth does not Granger cause financial distress in the sub-samples covering the period from July 2009 to November 2013. Moreover, the p-values in Figure 14 of an asymptotic Wald test implemented for a rolling window analysis suggest a unidirectional causality from monthly FSI, composed of 4 sub-indexes (bond, equity, foreign exchange and banking), to monthly IP growth, since we can reject the null hypothesis of non-causality in most of the sub-samples (see panel A). In line with the full sample analysis the only case associated with no evidence of causality in both directions is when we focus on the relationship between monthly CLIFS index by Duprey et al. (2017) and monthly IP growth. More specifically, the rolling window analysis results in Figure 15 suggest no causality between monthly CLIFS index by Duprey et al. (2017) and monthly IP growth.

7 Conclusions

In this paper, first, we construct a daily Financial Stress Index (FSI) for Lithuania. In particular, we extend the monthly Financial Stress Index (FSI) for Lithuania, computed by ECB (see Duprey et al. (2017)), to a high-frequency (daily) horizon and, given the important role played by a banking sector in the Lithuanian economic development in the recent decade, we include the banking sector among its constituents (beyond bond, equity, foreign exchange markets).
Moreover, we investigate the causal relationships between the daily FSI for Lithuania and monthly industrial production growth, using a Granger causality test applied to a mixed-frequency VAR characterised by a large mismatch in sampling frequencies of the series involved (i.e. daily vs monthly). The empirical findings suggest that the daily Lithuanian FSI has a predictive power for monthly Lithuanian IP growth for the full sample period (October 2001 – December 2016), but not vice versa. These results are also confirmed by a rolling-window analysis, where we allow the causal relationship to vary over time. Moreover, our analysis is in line with the empirical studies (see Hakkio et al. (2009), among the others) showing that an increase in financial stress has an adverse impact on the overall economic activity. In particular, we show that a banking stress in Lithuania leads to a stronger decline in IP growth.

Finally, we show that our daily FSI for Lithuania is a better predictor for monthly industrial production growth than a monthly Country-Level Indicator of Financial Stress developed by ECB (Duprey et al. (2017)).

Overall, our findings for Lithuania suggest that the leading indicator properties of an FSI index for Lithuania with respect to industrial production growth can be enhanced if we take into account the banking sector and we use daily observations.

Acknowledgements We would like to thank prof. Alain Hecq for many useful comments, suggestions and discussions.

References


A Appendix

A.1 Figures: stylized facts for Lithuania

Fig. 1: The development of Lithuanian financial system 2001-2016 (assets as % of GDP)

![Graph of Lithuanian financial system 2001-2016](https://www.lb.lt/en/financial-stability#ex-1-1).


Fig. 2: Lithuanian banking sector structure by assets (end-2016)


A.2 Figures: FSI for Lithuania

Fig. 3: Our REER in comparison with the REER for Lithuania from BIS

Notes: The straight line is the monthly REER index we compute by aggregating daily data. The dotted line is the monthly REER index available from BIS.

Fig. 4: Sub-indexes (2001-2016)
Fig. 5: Contribution of the sub-indexes to the overall FSI

![Fig. 5: Contribution of the sub-indexes to the overall FSI](image1)

**Notes:** We use EWMA with smoothing parameter 0.85 (see eq. 15) to estimate the pairwise correlations among sub-sectors.

Fig. 6: Time varying cross-correlations between the sub-indexes

![Fig. 6: Time varying cross-correlations between the sub-indexes](image2)

**Notes:** We use EWMA with smoothing parameter 0.85 (see eq. 15) to estimate the pairwise correlations among sub-sectors.

Fig. 7: Daily financial stress index for Lithuania

![Fig. 7: Daily financial stress index for Lithuania](image3)

**Notes:** Bars are associated with the following crisis events: the bankruptcy of US investment bank Lethman Brothers, first and second Greece bailouts, and SNORAS and Ukio bankas bankruptcy.
Fig. 8: Comparison between FSI (dark colour) and a CLIFS for Lithuania from ECB

Notes: The bold line is daily Lithuanian FSI aggregated into monthly frequency. The grey line is the monthly Lithuanian CLIFS index available from ECB at: https://sdw.ecb.europa.eu/browse.do?node=9693347.

Fig. 9: Lithuanian industrial production growth and FSI for Lithuania

Notes: In PANEL (A) we show the daily FSI, while in PANEL (B) the index is aggregated to monthly frequency. The IP growth is estimated as 12th month difference in log output, for a period October 2001 - December 2016. IP growth is in black colour and FSI is in red colour.
A.3 Figures: full sample analysis

**Fig. 10**: *Daily FSI (4 sub-indexes): the $\beta_i$ coefficient values in each $i^{th}$ model*

*Notes*: The plot shows the point estimates of the coefficients $\beta_i$ for each $i^{th}$ parsimonious model (see eq. 24) and the confidence bands (calculated as: Estimate $\pm 2 \times$ Str.Error).

**Fig. 11**: *Daily FSI (composed of 3 sub-indexes): the $\beta_i$ coefficient values in each $i^{th}$ model*

*Notes*: The plot shows the point estimates of the coefficients $\beta_i$ for each $i^{th}$ parsimonious model (see eq. 24) and the confidence bands (calculated as: Estimate $\pm 2 \times$ Str.Error). We consider a *daily FSI* constructed by aggregating only 3 sub-indexes: bond, equity and foreign exchange.
A.4 Figures: rolling window analysis

Fig. 12: P-values for rolling window analysis (daily FSI composed of 4 sub-indexes)

Note: We test for Granger causality between daily FSI for Lithuania (constructed in section 4) and monthly IP growth. Each test is based on MF-VAR (2) model and implemented by using a rolling window of 84 months (i.e. seven years). The x axes represent the 100 sub-samples, where the first sub-sample covers the period from October 2001 to September 2008, the second sub-sample covers the period from November 2001 to October 2008 and the last sub-sample covers January 2010 - December 2016 (the last observation of each sub-sample period is shown on the x axis). The y axes represent the p-values. The significance level of 10% is indicated by a light grey line.

Fig. 13: P-values for rolling window analysis (daily FSI composed of 3 sub-indexes)

Note: We consider a daily FSI for Lithuania constructed by aggregating only 3 sub-indexes (bond, equity and foreign exchange). Each test is based on MF-VAR (2) model and implemented by using a rolling window of 84 months (i.e. seven years). The x axes represent the 100 subsamples, where the first covers the period from October 2001 to September 2008, the second one from November 2001 to October 2008 and the last one January 2010 - December 2016. The y axes represent the p-values. The significance level of 10% is indicated by a light grey line.
Fig. 14: P-values for rolling window analysis (monthly FSI composed of 4 sub-markets)

Note: We consider an asymptotic Wald test between common frequency variables: a daily FSI for Lithuania (composed of 4 sub-indexes) aggregated to monthly frequency and monthly IP growth. An optimal lag length for each sub-sample is obtained by using a SC criteria. Each test is implemented by using a rolling window of 84 months (i.e. seven years). The x axes represent the 100 subsamples, where the first covers the period from October 2001 to September 2008, the second one from November 2001 to October 2008 and the last one January 2010 - December 2016. The y axes represent the p-values. The significance level of 10% is indicated by a light grey line.

Fig. 15: P-values for rolling window analysis (CLIFS index for Lithuania by Kluas et al., 2017)

Note: We consider an asymptotic Wald test between monthly CLIFS index by ECB and monthly IP growth, implemented by using a rolling window of 84 months (i.e. seven years). An optimal lag length for each sub-sample is obtained by using a SC criteria. The x axes represent the 84 subsamples, where the first covers the period from February 2003 to January 2010, the second one from March 2003 to February 2010 and the last one January 2010 - December 2016. The y axes represent the p-values. The significance level of 10% is indicated by a light grey line.
### Table 1: Indicators used for the Lithuanian FSI construction

<table>
<thead>
<tr>
<th>Market</th>
<th>Indicator</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond market</td>
<td>Realized daily volatility of 10y Lithuanian government bond yields</td>
<td>$V_{rR10_t}$</td>
</tr>
<tr>
<td></td>
<td>Cumulative difference of Lithuanian and German 10y bond yields</td>
<td>$CDIFF_t$</td>
</tr>
<tr>
<td>Equity market</td>
<td>Realized daily volatility of OMXV</td>
<td>$VOMXV_t$</td>
</tr>
<tr>
<td></td>
<td>The cumulative maximum loss (CMAX) of OMXV</td>
<td>$CMA_{XV_t}$</td>
</tr>
<tr>
<td>Foreign exchange market</td>
<td>Realized daily volatility of REER</td>
<td>$VREER_t$</td>
</tr>
<tr>
<td></td>
<td>Cumulative change over six months of REER</td>
<td>$CUMUL_{t}$</td>
</tr>
<tr>
<td>Banking sector: Swedbank</td>
<td>Realized volatility of the idiosyncratic Swedbank stock price returns</td>
<td>$VBKS_{Swed,t}$</td>
</tr>
<tr>
<td></td>
<td>CMAX of Swedbank stock prices</td>
<td>$CBKS_{Swed,t}$</td>
</tr>
<tr>
<td>Banking sector: SEB bank</td>
<td>Realized volatility of the idiosyncratic SEB bank stock price returns</td>
<td>$VBKS_{SEB,t}$</td>
</tr>
<tr>
<td></td>
<td>CMAX of SEB bank stock prices</td>
<td>$CBKS_{SEB,t}$</td>
</tr>
<tr>
<td>Banking sector: DnB bank</td>
<td>Realized volatility of the idiosyncratic DnB stock price returns</td>
<td>$VBKS_{DnB,t}$</td>
</tr>
<tr>
<td></td>
<td>CMAX of DnB bank stock prices</td>
<td>$CBKS_{DnB,t}$</td>
</tr>
</tbody>
</table>

Notes: all data is in real terms.

### Table 2: Lithuanian’s major trading partners, market share in %

<table>
<thead>
<tr>
<th></th>
<th>EZ</th>
<th>EE</th>
<th>LV</th>
<th>CN</th>
<th>CZ</th>
<th>DK</th>
<th>JP</th>
<th>NO</th>
<th>PL</th>
<th>RU</th>
<th>SE</th>
<th>TR</th>
<th>US</th>
<th>GB</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LT$</td>
<td>41.6</td>
<td>2.9</td>
<td>6.8</td>
<td>4.3</td>
<td>2.1</td>
<td>2.9</td>
<td>0.7</td>
<td>1.6</td>
<td>9.5</td>
<td>7.6</td>
<td>4.1</td>
<td>0.9</td>
<td>3.2</td>
<td>3.1</td>
<td>91.4</td>
</tr>
<tr>
<td>$LT_{adj}$</td>
<td>45.5</td>
<td>3.2</td>
<td>7.5</td>
<td>4.7</td>
<td>2.3</td>
<td>3.1</td>
<td>0.8</td>
<td>1.8</td>
<td>10.4</td>
<td>8.4</td>
<td>4.5</td>
<td>1.0</td>
<td>3.5</td>
<td>3.4</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: based on trade in 2008 – 2010. EZ – Eurozone (Austria 1.4, Belgium 4.0, Finland 2.7, France 4.8, Denmark 17.1, Ireland 0.3, Italy 5.3, Luxembourg 0.2, Netherlands 3.8, Portugal 0.3, Spain 1.7); EE – Estonia, LV – Latvia CN – China, CZ – Czech Republic, DK – Denmark, JP – Japan, NO – Norway, PL- Poland, RU-Russia, SE- Sweden, TR – Turkey, US- United States, GB- United Kingdom. Data is taken from BIS.
### Table 3: P-values for Granger Causality test (full sample analysis)

#### Panel (A): Daily FSI (including 4 sub-indexes) and monthly IP growth

<table>
<thead>
<tr>
<th>Test</th>
<th>Wald test (bootstrap version)</th>
<th>max-test</th>
<th>Wald test (bootstrap version)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF-VAR(1)</td>
<td>0.001</td>
<td>0.004</td>
<td>0.233</td>
</tr>
<tr>
<td>MF-VAR(2)</td>
<td>0.026</td>
<td>0.008</td>
<td>0.127</td>
</tr>
<tr>
<td>MF-VAR(3)</td>
<td>0.071</td>
<td>0.013</td>
<td>0.394</td>
</tr>
</tbody>
</table>

#### Panel (B): Daily FSI (3 sub-indexes) and monthly IP growth

<table>
<thead>
<tr>
<th>Test</th>
<th>Wald test (bootstrap version)</th>
<th>max-test</th>
<th>Wald test (bootstrap version)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF-VAR(1)</td>
<td>0.001</td>
<td>0.006</td>
<td>0.236</td>
</tr>
<tr>
<td>MF-VAR(2)</td>
<td>0.009</td>
<td>0.014</td>
<td>0.314</td>
</tr>
<tr>
<td>MF-VAR(3)</td>
<td>0.080</td>
<td>0.019</td>
<td>0.472</td>
</tr>
</tbody>
</table>

#### Panel (C): Monthly FSI (4 sub-indexes) and IP growth

<table>
<thead>
<tr>
<th>Test</th>
<th>Wald test (bootstrap version)</th>
<th>Wald test (asymptotic)</th>
<th>Wald test (bootstrap version)</th>
<th>Wald test (asymptotic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF-VAR(1)</td>
<td>0.001</td>
<td>0.000</td>
<td>0.876</td>
<td>0.850</td>
</tr>
</tbody>
</table>

#### Panel (D): Monthly ECB Country-Level FSI (3 sub-indexes) and IP growth

<table>
<thead>
<tr>
<th>Test</th>
<th>Wald test (bootstrap version)</th>
<th>Wald test (asymptotic)</th>
<th>Wald test (bootstrap version)</th>
<th>Wald test (asymptotic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF-VAR(2)</td>
<td>0.209</td>
<td>0.181</td>
<td>0.174</td>
<td>0.140</td>
</tr>
</tbody>
</table>

**Notes:** In panel (A) and (B) we investigate the causality between daily and monthly series. For this, we fit a MF-VAR specification with 1, 2, 3 lags and we use a max-test by Ghysels et al. (2018) and bootstrap version of Wald test by Götz et al. (2016). In panel (C) and (D) we test for Granger causality between two monthly series. For this purpose, we fit a common-frequency VAR model, where the optimal lag length is chosen by using a Bayesian information criteria and we use an asymptotic and a bootstrap version of a Wald test.
Table 4: Regression results (full sample analysis)

<table>
<thead>
<tr>
<th>Panel (A): monthly FSI (4 markets)</th>
<th>Panel (B): monthly CLIFS (by Duprey et al. (2017))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>0.043 (0.008)**</td>
</tr>
<tr>
<td>IP&lt;sub&gt;t-1&lt;/sub&gt;</td>
<td>0.520 (0.059)**</td>
</tr>
<tr>
<td>FSI&lt;sub&gt;t-1&lt;/sub&gt;</td>
<td>-0.236 (0.049)**</td>
</tr>
<tr>
<td>IP&lt;sub&gt;t-2&lt;/sub&gt;</td>
<td>0.115 (0.075)</td>
</tr>
<tr>
<td>FSI&lt;sub&gt;t-2&lt;/sub&gt;</td>
<td>0.039 (0.085)</td>
</tr>
</tbody>
</table>

Signif. codes: 0 : *** 0.001 : ** 0.01 : * 0.05

Notes: panel (A) present the point estimates for the coefficients in \( IP_t = \text{const}_t + a \times IP_{t-1} + b \times FSI_{t-1} + u_t \) and panel (B) for the coefficients in \( IP_t = \text{const}_t + \sum_{k=1}^{2} a_k \times IP_{t-k} + \sum_{k=1}^{2} b_k \times CLIFS_{t-k} + u_t \).

Table 5: Q test (full sample analysis)

<table>
<thead>
<tr>
<th>Lag order (p, pm):</th>
<th>k = 1</th>
<th>k = 5</th>
<th>k = 18</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FSI with 4 sub-sectors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MF-VAR (1)</td>
<td>0.624</td>
<td>0.887</td>
<td>0.035</td>
</tr>
<tr>
<td>MF-VAR (2)</td>
<td>0.987</td>
<td>0.981</td>
<td>0.119</td>
</tr>
<tr>
<td>MF-VAR (3)</td>
<td>0.796</td>
<td>0.68</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Notes: We find that a model with \( p = 2 \) and \( pm = 36 \) is better specified in terms of absence of residual correlation than other models taken under the consideration. In fact, when the model is fitted with \( p = 2 \) and \( pm = 36 \) we cannot reject the null hypothesis of no serial correlation in the residuals at 5% significance level, since the p-values of the test are \{0.987, 0.981, 0.119\}, respectively, for each lag \{1, 5, 18\}. 